Épijournal de Géométrie Algébrique – (yyyy), — submitted dd/mm/yyyy, accepted dd/mm/yyyy, revised dd/mm/yyyy

Towards a proof of the Hodge conjecture, and cycle spaces in positive characteristic

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in the memory of William V.D. Hodge

Abstract. An English abstract is suggested and should be descriptive enough by itself. Please do not include citations, footnotes or references to numbered equations, theorems, figures or tables in your abstract. Avoid complicated formulae or displayed equations.

Keywords. arithmetic functions, primitive roots, cyclotomic fields, Dirichlet series, multiplicative functions.

2010 Mathematics Subject Classification. 11R18, 11R32, 11M41, 11L07, 11N37, 11N45

Contents

1. Introduction

First of all, we want to stress that this paper has been typeset with the wonderful system TEX created by Don Knuth [Knu84], in the form later adapted by Leslie Lamport [Lam94], the current version of which is LATEX 2_ε(older versions won't work).

The present text serves as an example to show how the `epimath` style works (using the class file `epiarticle.cls` and the more specific style `epimath.sty`).

This style should be suitable for direct printing on A4 or Letter paper sheets without much change. When Letter paper format is used, the text will be more adequately centered if you specify `\setpapertype{letter}` at the beginning. The name of the particular epijournal (supposedly, from the Episciences project) is set by using `\journal{...}` before `\begin{document}`, namely `\journal{Mathematica Universalis}` here.

All data preceding the Introduction (Abstract, Keywords, MSC classification, table of contents should be enclosed by `\begin{prelims} ... \end{prelims}`. In case you'd like to change the keyword **Abstract** into something else, e.g. **Summary**, use `\def\abstractname{Summary}` just before entering `\abstract{...}`. Similarly with `\def\keywordsname{...}` to be used before entering `\def\keywords{...}`, or `\def\MSCclassname{...}`.

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We suggest to put the list of author addresses at the end (i.e., usually, after the bibliography list), by using `\authoraddresses` there.

As you may have observed, the use of a reference like `\cite{Lamport}` given in the bibliography list automatically creates an hyperlink to the list, and also shows at the end of the corresponding item in the bibliography list the number of all pages where the reference is mentioned – also with hyperlinks to facilitate navigation back and forth.

As far as mathematics are concerned, our main reference is [Gro68] in what follows.

2. Preliminary results

The following basic property will be used systematically.

Property 2.1. *...*

3. Fundamental results about motives

3.A. A new fonctor

We start with a few basic facts that will be useful in the sequel. The first paragraph in a section, subsection or subsubsection is normally not indented (as you may guess, these are introduced by typesetting `\section{...}`, `\subsection{...}`, `\subsubsection{...}` respectively). You can force indentation by putting `\forceindent` before. We did so here.

However, if you start a new paragraph in the course of a continued exposition, The new paragraph will appear to be indented, unless you specify `\noindent` before.

By default, all statement environments are set so as to use the same counter for theorems, propositions, lemmas, to avoid the possibly confusing situation where different types of statements get the same numbering. We recommend to respect this rule for any new environment to be created. You can e.g. specify `\newtheorem{assertion}[theorem]{Assertion}` to define the new environment "assertion", that will use the same counter as "theorem".

You may wish to use a vertical skip to indicate a new logical step, otherwise there will be only a new paragraph without any vertical skip. Here we use `\smallskip`.

Our main mathematical statement is

Theorem 3.1. *Let $\mathcal{F}$ be a fonctor from the category of algebraic schemes to the derived category of A-modules. (...)*

Proof. The proof is a bit long, so we first present a short sketch. Surprisingly, the cohomology calculations require the famous Rogers-Ramanujan identity

$$G(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q;q^5)_\infty (q^4;q^5)_\infty} = 1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 3q^6 + \cdots, \qquad (3.1)$$

where $(\bullet\,;\bullet)_n$ denotes as usual the q-Pochhammer symbol

$$(a;q)_n = \prod_{k=0}^{n-1} (1 - aq^k) = (1-a)(1-aq)(1-aq^2)\cdots(1-aq^{n-1}), \qquad (a;q)_0 = 1.$$

Another step uses the dilogarithm function

$$\mathrm{Li}_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}. \qquad (3.2)$$

Our arguments involve some preliminary steps which we call "first case" and "sporadic cases", as detailed below.

Remark 3.2. We recommend Theorems, Propositions and Lemmas to be typeset in italics, Remarks and Examples in roman characters.

3.A.a. First case

We proceed here by induction on dimension.

Initialization. Assume $n = 1$.

Inductive step. Assume the result known up to dimension $n - 1$ with $n \geq 2$.

3.A.b. Sporadic cases

We settle the sporadic cases of Theorem 3.1 by means of Formulae (3.1) and (3.2).

3.B. More about *l*-adic technology

We give here some technical facts about l-adic cohomology, along the lines of standard conjectures stated in [Gro68].

Proposition 3.3. *Let $\mathcal{R}$ be a representable fonctor ...*

Proof. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Sed non risus. Suspendisse lectus tortor, dignissim sit amet, adipiscing nec, ultricies sed, dolor. Cras elementum ultrices diam. Maecenas ligula massa, varius a, semper congue, euismod non, mi. Proin porttitor, orci nec nonummy molestie, enim est eleifend mi, non fermentum diam nisl sit amet erat. Duis semper. Duis arcu massa, scelerisque vitae, consequat in, pretium a, enim. Pellentesque congue. Ut in risus volutpat libero pharetra tempor. Cras vestibulum bibendum augue. Praesent egestas leo in pede. Praesent blandit odio eu enim. Pellentesque sed dui ut augue blandit sodales. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aliquam nibh. Mauris ac mauris sed pede pellentesque fermentum. Maecenas adipiscing ante non diam sodales hendrerit.

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(Notice here again the use of `\smallskip` to split two logically distinct parts of the explanations in the "proof"!)

3.C. Diagrammatic picture of the proof

The reader will understand better our strategy by looking at the following diagram. At the same time, this provides an example of how to include an EPS or PDF figure. With `dvips`, one must include encapsulated postscript files (EPS), while `pdflatex` needs JPG, PNG or (preferably) PDF format (search for `Flowchart.pdf` in the present source file).

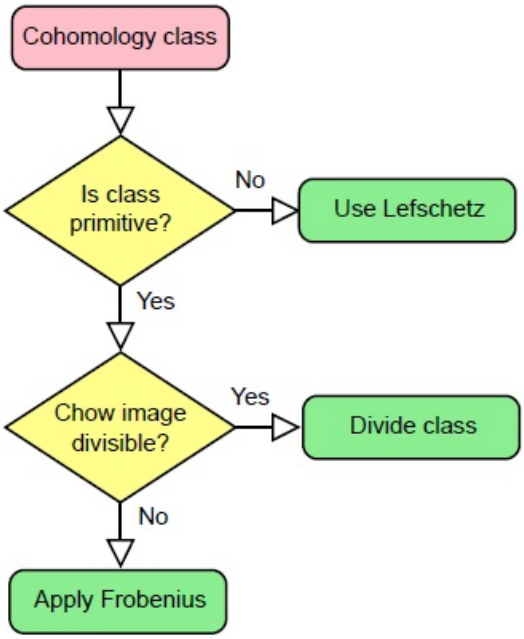

Figure 1: *Flow chart of proof*

4. Main arguments

The idea is to use a double induction on the weight and coniveau of the involved Hodge structures.

Lemma 4.1. *Let X be a nonsingular projective scheme over an algebraically closed field k of characteristic* 0. (...)

...
...
...

5. Further comments

In case the manuscript title is very long (or the list of authors is very long), the header line of odd pages might not have enough space to include this information. You can adjust this by specifying `\titlemark` and `\authormark`. Similarly, for titles of sections that would be too long to fit on headers of even pages, use `\sectionmark`.

The `epimath` latex style is otherwise pretty standard. The font size and line spacing of the list of references can be adjusted: it is recommended to use `\bibliographysize{small}` for a more compact presentation. The default heading "References" can be changed, e.g. to "Bibliography", by setting before `\renewcommand{\refname}{Bibliography}`. Also, a command like `\bibliographymark{References}` set before the bibliography list will let "References" appear in the right page headers instead if the last section – this can be useful if the bibliography list is very long.

Finally, `epimath.sty` provides special macros `\DOIstring{...}` to set the DOI on top of the first page, as well as `\ARXIV{...}`, `\HAL{...}` and `\MR{...}` to specify references to the preprint archives *arXiv* and *HAL*, and to *Math Reviews*, respectively, with appropriate hyperlinks. See e.g. [Voi11] and [Voi13] below.

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